A set of similar figures can be drawn for backward-facing waves. Figure 4 d) is then unchanged. The others are produced by reflection in the vertical axis, except that appropriate adjustments need to be made depending upon the magnitude and sign of  $u_0$ .

The essential point of the following discussion is that states connected by shocks or by simple waves to an arbitrary state  $(p_0, u_0)$ must lie on fixed curves given by eqs. (32), (34), (35), (36), provided

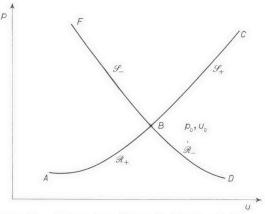


Fig. 5. – Wave transitions in the (p, u) plane.

there are no time-dependent terms in the constitutive relations. This is illustrated in Fig. 5. AB is represented by eq. (35), BC by (32), BF by (34) and BD by (36). AB and BC have a second-order contact at B, arising from the second order contact between isentrope and Hugoniot at the initial state  $(p_0, V_0)$ . So do BD and BF.

Some applications of the foregoing principles are illustrated in Fig. 6-13.

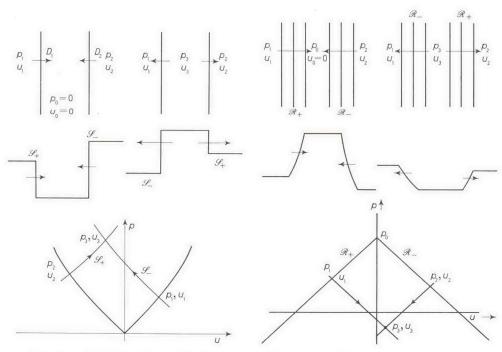


Fig. 6. - Collision of two shocks.

Fig. 7. - Collision of two rarefactions.

Figure 6: Two shocks approach one another, separated by a uniform state  $p_0 = 0 = u_0$ . They are represented by the points  $(p_1, u_1)$  and  $(p_2, u_2)$  in the (p, u) plane. After collision there are two waves drawing apart, separated by a new state  $(p_3, u_3)$  and running into the uniform states  $(p_1, u_1)$  and  $(p_2, u_2)$ , respectively. The transition from  $(p_2, u_2)$  to  $(p_3, u_3)$  occurs across a forward-facing wave, that from  $(p_1, u_1)$  across a backward-facing wave. The

 $p_1, u_1$  $p_0, u_0$  $p_2, u = 0$  $p_1, u_1$ 

Fig. 8. – Reflection of a uniform shock at a rigid wall.  $p_2/p_1 > 2$  if  $d^2p/dV^2 > 0$ .

state between the separating waves must be uniform in (p, u) because of continuity conditions on p and u. All these conditions are satisfied by the intersection of the  $\mathscr{S}_{-}$  and  $\mathscr{S}_{+}$  curves from  $(p_1, u_1)$  and  $(p_2, u_2)$ , respectively.

Figure 7: Collision of two rarefactions: The argument is analogous to the previous one. The initial state  $p_0$ ,  $u_0 = 0$ , is connected by rarefactions to  $(p_1, u_1)$  and  $(p_2, u_2)$ . The final state reached from  $(p_1, u_1)$  and  $(p_2, u_2)$  must lie at the intersection of wave transition curves passing through these states and must be a state of uniform p and q. This is satisfied by  $(p_3, u_3)$ .

Figure 8: Reflection of a uniform shock at a rigid wall: The initial shock,  $\mathcal{S}_+$ , carries material from  $(u_0 = 0 = p_0)$  to  $(p_1, u_1)$ . After reflection the state  $(p_1, u_1)$  is connected to the final state  $(p_2, u_2 = 0)$  by a backward-facing wave. The final state must lie on the curve for backward-facing transitions in the (p, u) plane and also on the u = 0 axis. These conditions are satisfied at point A. Note particularly that if the  $\mathcal{S}_+$  curve is concave upward in the (p, u) plane,  $p_2 > 2p_1$ . The  $\mathcal{S}_-$  curve through B is the mirror image of the  $\mathcal{S}_+$  curve in a vertical axis through B.

Figure 9: Reflection of a uniform shock at a free surface: This is similar to the preceding, but the final state must now lie on the p=0 axis. For condensed